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Why We Do Not See Fermion Parity Doublets

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ABSTRACT

Renormalization group arguments are applied to a Reggeon field theory which couples fermion and pomeron trajectories. The bare (input) fermion propagator has both positive and negative parity poles on the physical angular momentum plane for all values of u. However, after interacting with the Pomeron, the negative parity pole of the renormalized (output) fermion propagator migrates onto an unphysical angular momentum sheet for positive u, and so only the positive parity state appears as a physical particle. This provides a dynamical explanation for the disappearance of the negative parity partner, and is a property which is expected to be possessed by most reasonable Reggeon field theories.

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The application of Regge theory to processes involving Fermion exchange has led to some apparent inconsistencies. In the study of backward π-N scattering, one finds a symmetry relation, the MacDowell symmetry, among the various invariant amplitudes. This symmetry assumes the absence of dynamical singularities at u=0 in the A and B amplitudes. If one describes this process by the exchange of fermion Regge poles, the MacDowell symmetry implies that the trajectories of the positive and negative parity fermions are related: $\alpha_+(W) = \alpha_-(-W)$ where $W = \sqrt{u}$. On the other hand, the nucleon trajectory appears to be linear in u, and so it would seem that there should exist negative parity nucleon particles which are nearly degenerate with their positive parity partners. The failure to observe these negative parity states has for many years been an outstanding phenomenological puzzle.

Using the Van Hove model, Carlitz and Kislinger[1] showed that one could construct a more or less acceptable amplitude for backward π-N scattering by assuming *ab initio* that only positive parity particle states exist. Their procedure was to sum, *a la* Van-Hove, an infinite sequence of zero-width resonances, each one of which has positive parity when placed on its spin shell. Attempts have also been made to understand the absence of parity partners in dual theories.[2] Despite these efforts, no fully successful dynamical mechanism has been proposed to account for the disappearance of the parity doublets.

In this note, we wish to describe the results of a calculation which shows dynamically why negative parity partners do not appear as physical particles. We study the interaction of the pomeron with a pair of fermions of opposite parity in a Reggeon field theory using the renormalization group. As a result of the pomeron-fermion interaction, the fermion propagator develops a cut and its negative parity pole disappears from the physical angular momentum sheet for positive u. Note that unlike the approach of Carlitz and Kislinger,[1] we make no a priori assumptions about the absence of negative parity particles. Nevertheless, the renormalized fermion propagator has only positive parity singularities on the physical angular momentum sheet for u > 0.

In what follows, we shall briefly describe the Reggeon field theory with which we deal, and then present the results which are important for our argument about the disappearance of the parity partners. A fuller description of this work, as well as a discussion of its implications for the Reggeon calculus will be presented elsewhere.[3]

The field theory we have studied has a pomeron whose bare propagator is

$$i\,[E-\alpha_0{'}k^2+i\epsilon]^{-1} \tag{1}$$

and a fermion with a bare propagator given by

$$i\,[E-\Delta_0+\beta_0\sqrt{u}+\alpha_F{'}u]^{-1}\Lambda_+ + i\,[E-\Delta_0-\beta_0\sqrt{u}+\alpha_F{'}u]^{-1}\Lambda_-$$

$$= i\,[E-\Delta_0-\beta_0\hat{k}+\alpha_F{'}u+i\epsilon]^{-1} \tag{2}$$

where $\Lambda_\pm = \frac{1}{2}(1 \mp \frac{\hat{k}}{k})$ is the projection operator on parity eigenstates, $E = 1 - j = 1$ - angular momentum, $k^2 = -k_\perp^2 = u$, $\hat{k} = -\gamma_\perp k_\perp$, and Δ_0 is a constant which describes the intercept of the bare trajectory. For large time-like u, the trajectories in (2), $\alpha_F(\sqrt{u}) = 1 - \Delta_0 \pm \beta_0' \sqrt{u} + \alpha_F' u$, behave like u, are nearly degenerate, and both of them contain particle poles. In order to establish the analytic structure needed to eliminate one of the parity partners, we want to determine the nature of the fully renormalized fermion propagator in the region of small $|u|$ and $|E - \Delta_0|$. In calculating the renormalization group functions which determine the infrared behavior of the Green's functions, only terms lowest order in $\sqrt{u}$ are important,[3,8] and so, for simplicity we set $\alpha_F' = 0$ when calculating Feynman graphs. Nevertheless, the bare fermions propagators are of the form (2), and both parity pieces contain particle poles.

The unrenormalized interactions are a triple pomeron coupling, λ_0, and a fermion-fermion-pomeron vertex, r_0. The couplings are chosen to be purely imaginary, as required for an absorptive pomeron. Mass counterterms are also included as needed, to keep the intercepts of the renormalized trajectories at their observed value.[4]

We are interested in this theory for small k^2, and E close to the trajectories. In that limit, the fermion does not significantly affect the pomeron, and so the pomeron renormalization problem decouples from the fermion. This pomeron problem has been considered by Abarbanel and Bronzan,[5] and the reader is referred to their paper for details.

In renormalizing the fermion, we dimensionally regulate our integrals and choose as a renormalization point some negative energy, $-E_N$, with all momenta set equal to zero. Our theory is scale invariant in one time (angular momentum) dimension, and D = 4 space dimensions. In the Reggeon field theory physics takes place when D=2. We therefore make an ϵ-expansion about D=4 (for us ϵ=2), and proceed to calculate the renormalization group functions to lowest non-trivial order in ϵ. In this theory there are three dimensionless renormalized parameters which we choose to be

$$g = \frac{\lambda}{E_N}\left(\frac{E_N}{\alpha'}\right)^{D/4}$$

$$h = \frac{r}{E_N}\left(\frac{E_N}{\alpha'}\right)^{D/4} \tag{3}$$

and

$$\rho = \frac{\beta'^2}{\alpha' E_N}$$

α' and β' are the renormalized pomeron and fermion slopes, respectively, λ is the renormalized triple pomeron coupling constant, and r is the renormalized fermion-fermion-pomeron coupling constant.

We can now solve the coupled renormalization group equations in the usual way, and search for fixed points in this three dimensional parameter space. Doing this, we find that there are two physically acceptable fixed points at which the infrared behavior is described by moving fermion and pomeron singularities and imaginary triple pomeron and fermion-fermion-pomeron couplings. In addition, there are several other fixed points of more mathematical and scholarly interest.[3] Interestingly, none of these fixed points is infrared stable. They are all unstable, and can be approached only from a plane in the three-dimensional parameter space.[7] Nevertheless, we can calculate the infrared behavior of the theory assuming that we are at the fixed point, or on the plane of approach. Let us now describe the behavior of the fermion propagator at the physically interesting fixed points.

In the limit that we scale $E - \Delta_0 \equiv F \rightarrow 0$, we can write for the renormalized inverse fermion propagator

$$\Gamma(F, k^2, \alpha', \rho, g, h, E_N)$$
$$= E_N \left(\frac{F}{E_N}\right)^{\gamma} \left\{ \phi_1 \left[\frac{\alpha' k^2}{E_N} \left(\frac{E_N}{F}\right)^{z}, \rho, g, h \right] + \left(\frac{\alpha'}{E_N}\right)^{\frac{1}{2}} \left(\frac{E_N}{F}\right)^{z/2} \hat{k}\, \phi_2 \left[\frac{\alpha' k^2}{E_N} \left(\frac{E_N}{F}\right)^{z}, \rho, g, h \right] \right\} \tag{4}$$

where γ is not an integer at $D = 2$, and $z = 1 + \frac{\epsilon}{24}$. The functions ϕ_1 and ϕ_2 are dimensionless scaling functions whose arguments must be dimensionless variables. The values of ρ, g, and h at the fixed points in question are non-zero constants. They do not enter into this discussion in an important way, and will be suppressed from now on. Notice that the only way in which F and k^2 enter into the scaling functions is in the combination $\propto k^2 F^{-z}$. This is exactly the same form as that found for the pomeron trajectory by Abarbanel and Bronzan.[5]

The right hand side of Eq. (4) can be written in a more convenient form:

$$\Gamma(F,k^2) = \Gamma_+(F,k^2) + \Gamma_-(F,k^2) = E_N\left(\frac{F}{E_N}\right)^{\gamma}\left[\phi_+(x)\Lambda_+ + \phi_-(x)\Lambda_-\right] \quad (5)$$

so that for the fermion propagator we have

$$G(F,k^2) = G_+ + G_- = \frac{1}{E_N\left(\frac{F}{E_N}\right)^{\gamma}\phi_+(x)}\Lambda_+ + \frac{1}{E_N\left(\frac{F}{E_N}\right)^{\gamma}\phi_-(x)}\Lambda_-$$

where

$$\Lambda_\pm = \frac{1}{2}\left(1 \mp \frac{\hat{k}}{k}\right)$$

$$\phi_\pm(x) = \phi_1(x) \pm x^{\frac{1}{2}}\phi_2(x)$$

and

$$x = \frac{\alpha' k^2}{E_N}\left(\frac{E_N}{F}\right)^z .$$

$\Lambda_+(\Lambda_-)$ is the projection operator for the positive (negative) parity part of the fermion propagator, and so $\Gamma_\pm$ are the inverse propagators for the positive and negative parity fermion states.

Now, suppose there is a zero of Γ_+ for some value of x, say x_0. Furthermore, suppose that

$$\phi_{1,2}(x_0) = \phi_{1,2}(e^{2\pi i n}x_0) \quad (6)$$

where n is any integer. (This is true in an ϵ-expansion of the scaling functions, as we shall discuss later.) In that case, it is easy to see that

$$\phi_+(x_0) = 0 \Rightarrow \phi_+(e^{4\pi i n}x_0) = \phi_-(e^{2\pi i(2n+1)}x_0) = 0 \qquad (7)$$

For fixed u, we can now look in the F-plane and ask where these other zeros of Γ_+ and Γ_- are. Writing $u = ye^{i\theta}$, we find that

$$\Gamma_+ = 0 \text{ when } F = Ce^{-i(4\pi n - \theta)/z}$$

and (8)

$$\Gamma_- = 0 \text{ when } F = Ce^{-i(4\pi n + \theta)/z}$$

where

$$C = E_N\left(\frac{\alpha' y}{x_0 E_N}\right)^{1/z} .$$

Because of the cut in the inverse propagators generated by the factor F^{γ}, these singularities will lie on different sheets of F. When $\theta = 0$, $u > 0$, and we can define the physical sheet to be that sheet on which the zero of Γ_+ lies for n=0. It is convenient to place the cut along the negative real F-axis. Using (8), it is then easy to see that there are no other zeros of Γ_+ or Γ_- on the physical sheet for positive u. This means that only the positive parity state is present as a physical particle.

We can now continue in u to a negative real value by letting $\theta \to \pi$. The motion of the n=0 zeros of Γ_+ and Γ_- is indicated in Fig. 1. Notice that the zero of Γ_- moves through the cut onto the physical sheet for $u < 0$. Hence, it contributes to backward π-N scattering, even though it does not exist as a physical particle. The other zeros of Γ_+ and Γ_- are all at least two sheets away, and for $0 \le \theta \le \pi$ do not appear on the physical sheet.

There are two very important ingredients for this result. First, the critical exponent, γ, must be non-integral so that there is a F-plane cut in the inverse propagators. This is likely to be a general feature of theories such as ours which are not infrared free, even when calculated to higher orders in ϵ. Second, Eq. (6) must be valid in order that we be able to determine the zeros of Γ_- given the zeros of Γ_+. To lowest order in ϵ, we can determine the scaling functions by observing that at all fixed points, $h \propto \epsilon^{\frac{1}{2}}$. This means that to lowest order in ϵ, the scaling functions can be simply determined from the bare propagator, and Eq. (6) can easily be seen to be valid. Moreover, an analysis of terms higher order in ϵ indicates that (6) is valid there also.[3] Note that we do not require an ϵ-expansion of the scaling functions for arbitrary values of x, but only for a certain finite value. This is fortunate, since the radius of convergences in ϵ, of the ϕ s may depend on x, especially when $x \to \infty$.[7] But this problem need not concern us.

The formula in Eq. (5) seems to indicate the presence of a fixed cut in the F-plane. However, our argument does not require that this cut be fixed. All that is necessary is that when θ is continued from π to zero, the zero of Γ_- passes through the cut onto a different sheet. The scaling functions could contain compensating singularities which cause the cut to move with u, but as long as it doesn't move too fast our conclusions are still valid. On the other hand, the scaling functions could, in principle, contain a cut which exactly cancels the factor F^{γ}. In that extreme case, of course, our arguments would be invalid. However, in view of the preceding discussions we consider this an unlikely possibility.

It is interesting to note that our renormalized trajectories no longer behave like $\sqrt{u}$ for small u but almost as u. One could, of course, have chosen bare fermion trajectories which, for small u, were proportional to u. However, the simplest starting point for our theory is to try to expand the bare trajectories in a Taylor's series in $\sqrt{u}$, as suggested by the MacDowell symmetry, and there is no a priori reason to drop the term proportional to $\sqrt{u}$. The exact behavior of the trajectories for larger u cannot easily be determined from our considerations. Nevertheless, regardless of what the form of the trajectories is for large, time-like u, our argument on the disappearance of the negative parity partner pole from the physical sheet will be valid, unless for large, timelike u, the pole turns around, moves through the cut and onto the physical sheet again. We cannot prove that this cannot appear but it is clearly an unnatural option.

In this note we have discussed the dynamical mechanism by which fermion parity partners are removed from the physical j-plane sheet for positive u, even though they are present in the bare unrenormalized theory. Notice that we do not require the absence of all negative parity fermions. Our result only indicates that those that appear cannot be though of as parity partners of other, observed, positive parity fermions. The requirements of the renormalization program which are necessary to achieve this effect are not very restrictive and are likely to be satisfied by most reasonable Reggeon field theories. One outstanding exception to this are certain infrared free theories in which an F-plane cut will not be induced.[9] But aside from such theories, it is difficult to imagine a reasonable renormalized world in which the fermion parity partners would not be forced to disappear.

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REFERENCES

[1]R. Carlitz and M. Kislinger, Phys. Rev. Lett. 24, 186 (1970).

[2]See, for example, K. Bardakci and M. B. Halpern, Phys. Rev. Lett. 24, 428 (1970).

[3]J. Bartels and R. Savit, FNAL-Pub-74/61-THY, submitted to Phys. Rev. D.

[4]A similar fermion-pomeron field theory with $\lambda_0 = 0$, was studied by V. N. Gribov, E. M. Levin and A. A. Migdal, Yad. Fiz. 11, 673 (1969). [Soviet Journal of Nucl. Phys. 11, 378 (1970).] However, our results are substantially different from theirs. See Ref. (3) for further details.

[5]H. D. I. Abarbanel and J. B. Bronzan, NAL-Pub-73/91-THY (1973).

[6]This instability can be traced to the fact that the pomeron and fermion slopes have different dimensions. See Ref. (3) for a more complete explanation. Stable fixed points do occur in other fermion theories

with a different pomeron. R. Savit, in preparation.

[7] We wish to thank A. R. White for discussions on this point.

[8] Higher order terms in the bare trajectory have been examined in pure pomeron theories and are found to be unimportant for the small t behavior. A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan, ITEP Preprint No. 102; R. Brower and J. Ellis, Preprint UCSC 74/101.

[9] Depending on one's taste, this could be used as an argument against infrared freedom, at least for the fermions. For a different point of view, see H. D. I. Abarbanel, NAL-Pub-74/42-THY.

FIGURE CAPTION

Fig. 1 Motion of the $n = 0$ positive and negative parity poles of the fermion propagator in the F-plane as u goes from a positive to a negative real value. The solid line refers to the physical sheet, while the dashed line refers to the unphysical sheet.

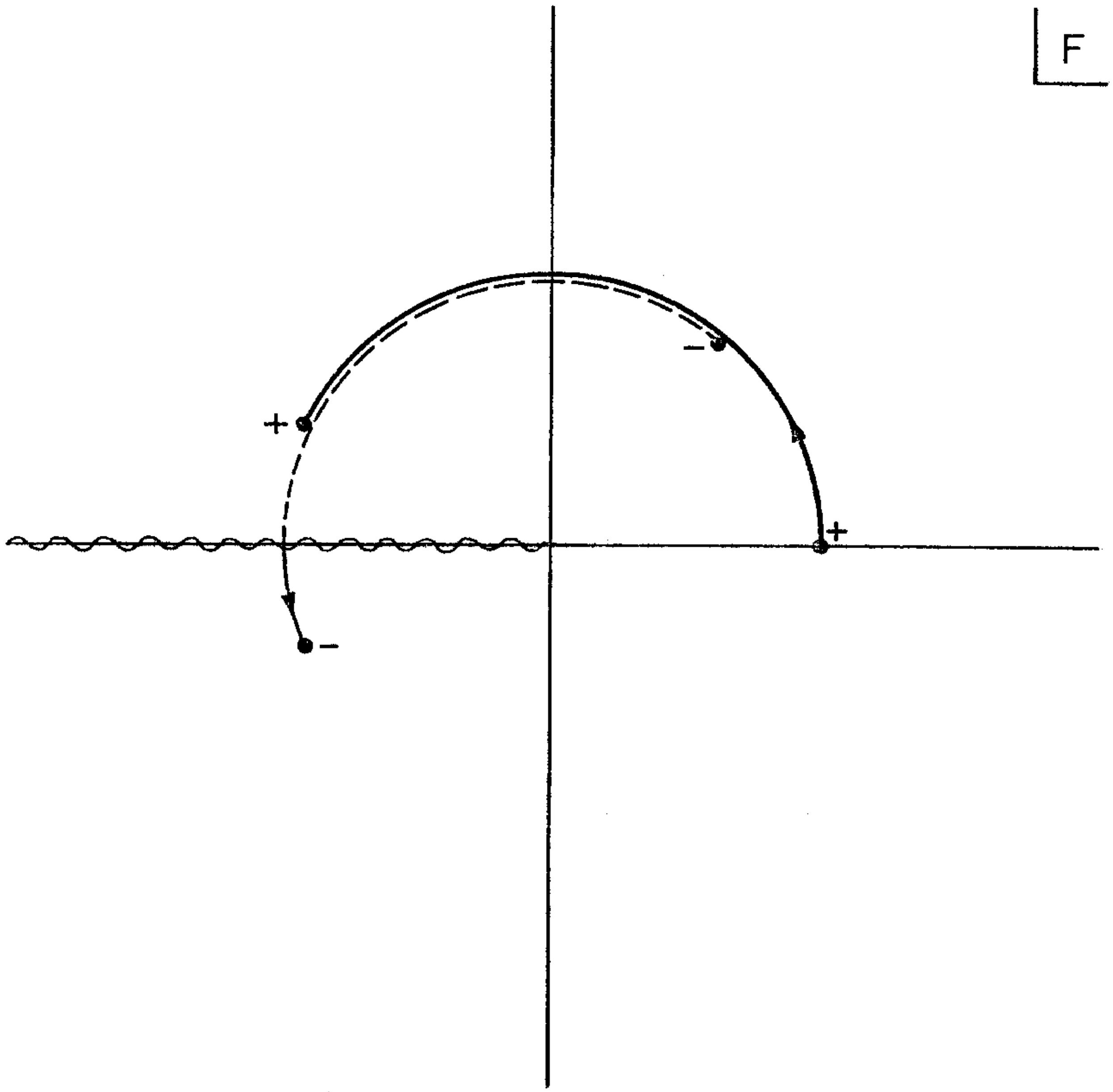

Figure 1